

# *Three dimensional modeling of the effects of different spray angles and number holes on the combustion process and emission formation in a direct injection diesel engine*

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Received Date: 6 Sept.; 2010

Accepted Date: 8 Dec.; 2010

## *Abstract*

This study presents a computational study of mixture formation, flame front movement, combustion process and emission formation at the various spray angles and number holes in a direct injection diesel engine. This work is carried out in a direct injection OM355 diesel engine at four and five holes with 130°, 135° and 140° spray angles. The amount of fuel injected per cycle is constant at all states. The results of the model show that with an increasing spray angle, the combustion parameters such as heat release rate, in cylinder pressure and temperature because of more spray impinging decrease while with an increasing number of holes, a slight variation take place at these parameters. From emission formation point of view, with an increase in spray angle, NO<sub>x</sub> decreases and soot increases. Also with an increase in the number of holes, the similar trends are observed. The results of model for baseline engine are good agreement with the corresponding experimental data. In other cases, the results are in good agreement with the corresponding data in the literature.

## *Keywords*

Diesel engine, direct injection, flow field, combustion process, spray angle, hole.

## **1. Introduction**

The internal combustion engine is by far the most important power train for all kind of vehicles today. Up to now there is no alternative to this kinds of engine, and it is for sure that it will keep its leading position for at least the next three to five decades. However, it has to be continuously improved, and great efforts have to be made in order to increase efficiency and to fulfill future emission legislation.

One method for achieving this goal and the reduction of engine-out raw emissions is apply improved or new mixing formation and combustion concepts that will be one of the key measures to keep the in-

ternal combustion engine up to date. Therefore, the exact numerical simulation and optimization of mixture formation and combustion processes is nowadays becoming more and more important. One advantage of using simulation models is that in contrast to the experiments, results can often be achieved faster and cheaper. Much more important is the fact that despite the higher uncertainty compared to the experiments, the numerical simulation of mixture formation and combustion processes can give much more extensive information about complex in-cylinder processes than the experiments could ever provide. Using numerical simulations, it is possible to calculate the temporal behavior of every variable

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of interest at any place inside the computational domain. This allows the attainment of a detailed knowledge of the relevant processes and is a prerequisite for their improvement [1]. There are three classes of models that can be used in numerical simulations of in-cylinder processes. If very short calculation times are necessary, so-called thermodynamic models are used. These zero-dimensional models, which do not include any spatial resolution, only describe the most relevant processes without providing insight into local sub-processes [1].

The second class of models are the phenomenological models, which consider some kind of quasi-spatial resolution of the combustion chamber and which use more detailed the sub-models for the description of the relevant processes like mixture formation, ignition and combustion[1]. The third class of models is the computational fluid dynamics (CFD) models [2-7]. In CFD codes, the most detailed sub-models are used, and every sub-process of interest is considered. For example, in case of mixture formation, the sub-processes injection, break-up and evaporation of single liquid droplets, collisions of droplets, impingement of droplets on the wall etc. are modeled and calculated for every individual droplet, dependent on its position inside the three-dimensional combustion chamber. Thus, this class of models is the most expensive regarding the consumption of computational power and time.

The CFD codes are especially suited for the investigation of three-dimensional effects on the in-cylinder processes, like the effect of tumble and swirl, the influence of combustion chamber geometry, position of injection nozzle, spray angle, number of holes, etc.

Therefore, at present work, the simultaneously effects of different spray angles and number of holes on the combustion process and emission formation are studied by CFD code.

## 2. Model Specification

The numerical method is used at present work, which is based on Fire CFD code .the engine considered is heavy duty OM-355 diesel engine. The engine specifications are given in table (1). The calculations are carried out for 1400 rev/min at full load state, in which the emission formation

reaches its maximum values. Calculations are carried out on a closed system from IVC at -118°C<sub>A</sub> to EVO at 120°C<sub>A</sub>. For baseline engine, injector is centric with four holes at baseline engine which is fixed at 3mm below cylinder head and spray cone angle is equal to 130°C<sub>A</sub>. swirl ratio for engine 1.1 is approximated.

Thermodynamic data for the material streams are given in Simulations are carried out at six cases, which are include four and five holes with 130, 135 and 140 spray angles. In all cases after start of injection, standard WAVE model is used for instability and spray breakup model and the mass injected per cycle are the same. Initial fuel injection pressure is 195bar with injection period starting from -18 ° CA and ending at 0° CA .Total area of the holes, start and end of injection, injection duration and injection rate modes are constant in all cases. Similar temperature and pressure values are chosen as initial conditions. It is assumed that the initial fuel droplets have the diameter of nozzle hole which is technically called blob injection.

Table 1. OM-355 Engine specifications

Engine type	Heavy duty D.I Diesel engine
Number of injector holes	4
Engine speed at max torque	1400rpm
Engine speed at max power	2200rpm
Piston diameter*stroke	150*128mm
Cylinder volume	11.58lit
Injection pressure	195 bar
Max out put power	240 hp
Max outlet torque	824N.m
Number of cylinders	6,vertical type
Compression ratio	16.1:1
Turbulence Kinetic Energy	18 m <sup>2</sup> /s <sup>2</sup>
Turbulence Length Scale	0.0075 m

Considering the symmetry of the model at four holes, the problem is only solved for a 90 degrees sector. Figure 1 shows the numerical grid with illustrated surface boundary conditions that include two periodic surfaces, head, liner and piston. The grid is designed to model the geometry of the engine and contains a maximum of 69,800 cells at 120°CA BTDC.

Equations used by numerical model are as follows [6]: continuum equation:

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{\partial}{\partial x_j}(\hat{\rho} \hat{U}_j) \quad (1)$$

$k - \mathcal{E}$  RNG turbulent and momentum equation:

$$\rho \frac{\partial k}{\partial t} + \rho U_j \frac{\partial k}{\partial x_j} = P + G - \mathcal{E} + \frac{\partial}{\partial x_j} \left( \mu + \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_j} \right) \quad (2)$$

$$\rho \frac{D\mathcal{E}}{Dt} = \left( C_{\mathcal{E}1} P + C_{\mathcal{E}3} G + C_{\mathcal{E}4} k \frac{\partial U_k}{\partial x_k} - C_{\mathcal{E}2} \mathcal{E} \right) \frac{\mathcal{E}}{k} + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_{\mathcal{E}}} \frac{\partial \mathcal{E}}{\partial x_j} \right) \quad (3)$$

$$P = -2\mu_t S : S - \frac{2}{3} [\mu_t (trS) + K] (trS) \quad (4)$$

$$G = -\frac{\mu_t}{\rho \sigma_{\rho}} \nabla \rho \quad (5)$$

$$\mu_t = C_{\mu} \rho \frac{k^2}{\mathcal{E}} \quad (6)$$

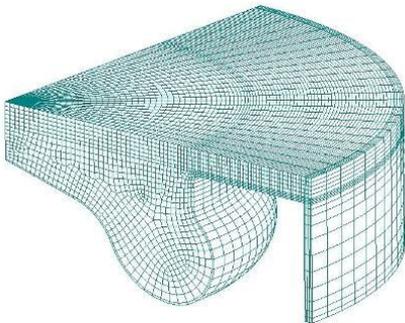


Fig.1: Computational mesh segment

With constants set by values below:

$C_{\mu}$	$C_{\mathcal{E}1}$	$C_{\mathcal{E}2}$	$C_{\mathcal{E}3}$	$C_{\mathcal{E}4}$	$\sigma_k$	$\sigma_{\mathcal{E}}$	$\sigma_{\rho}$
0.09	1.44	1.92	0.8	0.33	1	1.3	0.9

### 3. Energy Equation:

$$\rho \frac{DH}{Dt} = \rho \left( \frac{\partial H}{\partial t} + U_j \frac{\partial H}{\partial x_j} \right) = \rho \dot{q}_s + \frac{\partial P}{\partial t} + \frac{\partial}{\partial x_i} (U_j \tau_{ij}) + \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T}{\partial x_j} \right) \quad (7)$$

### 4. Spray model

Wave breakup model is used which is further modified to account for spray wall impingement effects, and is also improved by considering droplet distortion to obtain dynamically varying drop drag coefficients [6].

In this model the growth of an initial perturbation on a liquid surface is linked to its wave length and to other physical and dynamic parameters of the injected fuel and the domain fluid. The initial fuel droplets have the diameter of nozzle hole which is technically called blob injection. Droplet dissipation rate is modeled by the following equation:

$$\tau = \frac{3.726 \cdot C_2 \cdot r}{\Lambda \Omega} \quad (8)$$

In the above equation,  $\Lambda$  and  $\Omega$  are wave length and growth rate and are functions of droplet characteristics and critical Weber number. Also droplet radius is assumed to obey the equation at steady state. Detailed information can be found in reference [6 and 10].

$$r_{stable} = \min \left\{ \left( 3\pi^2 \cdot U / 2\Omega \right)^{0.33}, \left( 3r^2 \Lambda / 4 \right)^{0.33} \right\} \quad (9)$$

For prediction of impinging jet and heat transfer are used from standard wall function and wall-jet respectively.

## 5. Combustion rate EQUATION (combustion model)

This model assumes that in premixed turbulent flames, the reactants (fuel and oxygen) are contained in the same eddies and are separated from eddies containing hot combustion products. The rate of dissipation of these eddies determines the rate of combustion [6].

$$\bar{\rho} \dot{r}_{fu} = \frac{C_{fu}}{\tau_R} \bar{\rho} \min \left( \frac{\bar{y}_{fu}}{y_{fu}}, \frac{\bar{y}_{ox}}{S}, \frac{C_{pr} \bar{y}_{pr}}{1+S} \right) \quad (10)$$

The first two terms of the “minimum value of” operator determine whether fuel or oxygen is present in limiting quantity, and the third term is a reaction probability, which ensures that the flame is not spread in the absence of hot products.  $C_{fu}$  and  $C_{pr}$  are empirical coefficients and  $\bar{y}_{pr}$  is product mass fraction that which is includes intermediate species, CO<sub>2</sub> and H<sub>2</sub>O. The value of  $C_{fu}$  requires adjustment with respect to the experimental combustion data for the case under investigation.  $C_{fu}$  varies from 3 to 25 in diesel engines. An optimum amount of 8 for  $C_{fu}$  was selected according to the experimental data.  $\tau_R$  is the characteristic time for reaction turbulent mixing and is defined as below:

$$\tau_R = \frac{k}{\varepsilon} \quad (11)$$

This specifies the combustible mixture consumption rate.

### emission model

The thermal NO<sub>x</sub> formation mechanism is expressed in terms of the extended Zeldovich mechanism [11]:



Formation rate parameter for NO<sub>x</sub> formation is selected according to experimental data (as shown in figure 3).

The Hiroyasu and Magnussen [12] mechanism for soot formation rate is modeled as the difference between soot formation and soot oxidation:

$$\frac{dm_{soot}}{dt} = \frac{dm_{form}}{dt} - \frac{dm_{oxid}}{dt} \quad (15)$$

$$\frac{dm_{form}}{dt} = A_f m_{fv} P^{0.5} \exp \left( -\frac{E_a}{RT} \right)$$

with  $A_f$  as the pre exponential factor,  $m_{fv}$  is the fuel vapor mass,  $P$  is the pressure and  $E_a$  is the activation energy.

$$\frac{dm_{oxid}}{dt} = \frac{6M_c}{\rho_s d_s} m_s R_{tot} \quad (16)$$

where  $M_c$  is the carbon molecular weight,  $\rho_s$  is the soot density,  $d_s$  is the average soot diameter,  $M_s$  is the soot mass and  $R_{tot}$  is the net reaction rate.

Formation and oxidation rate parameter for soot emission are selected as those of defaulted in the software and according to experimental data (as shown in figure 4).

## 6. Ignition model

The Shell auto-ignition model was used for modeling of the auto ignition of diesel fuel [13]. In this generic mechanism, 6 generic species for hydrocarbon fuel, oxidizer, total radical pool, branching agent, intermediate species and products were involved. In addition the important stages of auto ignition such as initiation, propagation, branching and termination were presented by generalized reactions, described in [13].

## 7. Results and discussions

Figure 2 shows the comparison of predicted and experimental in-cylinder pressure for base-line engine with spray angle 135°CA and four holes at full load state with 1400rpm. The results presented in the figures are global (cylinder averaged) quantities as a function of time (crank angle). The peak pressures discrepancy between

experiment and computation is less than 1%. Also, it is assumed that injection rate to be constant and also it is clear from this figure that the start of combustion exactly can be predicted by both pressure and heat release rate curves. Figures 3 and 4 represent the comparison of model prediction and measured data [8] for NOx and Soot emissions and show good agreement which validates model for the further investigation. It is clear from these figures that the predicted NOx emission is over predicted and the soot emission exactly is predicted compared to experimental results by the model. The good agreement between measured and predicted data for the combustion process during the compression and expansion strokes verifies the results of the model. This verification demonstrates that multidimensional modeling can now be used to gain insight into the combustion process and emission and also to provide direction for exploring new engine concepts.

present work includes six states and in all six states, the injectors are mounted in the center of combustion chambers. These states are divided two parts that in the first part, the number of holes are four and five and in the other part, the spray angle is varied from 130° to 140° by 5° step.

Figures 5-7 shows the in-cylinder pressure, temperature and heat release rate traces with different spray angles and number of holes respectively. It can be seen that the peak values for the pressure, temperature and heat release rate are reduced with the increased spray angle and number of holes, hence can lead to reduce the indicated work per cycle and higher isfc (indicated specific fuel consumption). In all cases, the start and end of combustion (combustion duration) are the same. Also, it is interesting that in addition to combustion duration, ignition delay is not varied at different spray angles and number of holes.

As shown in these figures that one of the most important influence factor is the fuel The injection system. The design of the injection nozzle (e.g. number of holes, spray direction) must be carefully adjusted to the boundary conditions like combustion chamber geometry, air motion, and pressure inside the cylinder. For example, in the case of a strong contribution of air motion (swirl) to the mixture formation process, less noz-

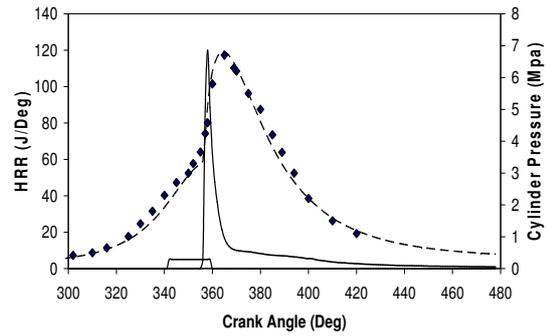


Fig.2: The comparison of CFD model and experimental Pressure data [8] for the baseline engine at 1400rpm

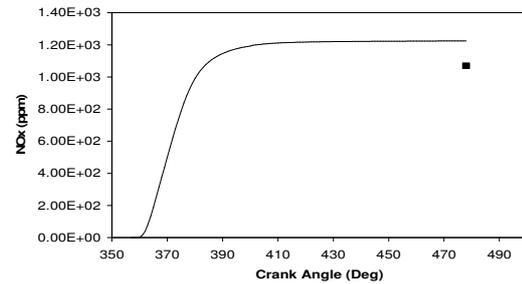


Fig.3: The comparison of CFD model and experimental exhaust NOx emission data [8] for the baseline engine at 1400rpm

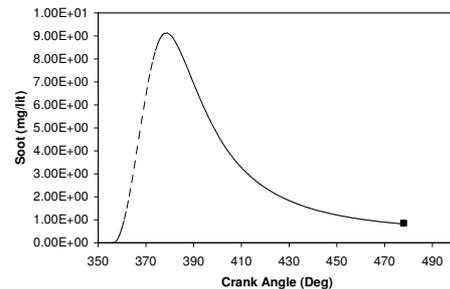


Fig.4: The comparison of CFD model and experimental exhaust Soot emission data [8] for the baseline engine at 1400rpm

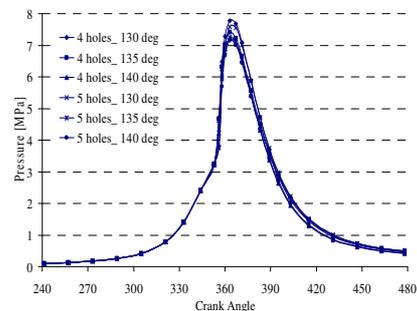


Fig.5: Comparison of pressure traces for different number of holes and spray angle at 1400rpm

zle holes and lower injection pressures are necessary than in the case of low-swirl combustion concepts[9].

The generation of strong swirl increases the pressure losses in the intake system and tends to increase fuel consumption. Further on, ignition delay and premixed peak are usually increased. If too many nozzle holes are used, the burning spray plumes may be displaced by the air motion in a way that fuel is injected in the burnt gases of the neighbor plume. This strongly increases soot formation. Today, low-swirl combustion concepts are often used, and the energy for mixture formation is more or less solely provided by the spray. For this reason, injection pressures and number of holes are increased, and wide piston bowls, which allow the necessary spray penetration in order to include the complete cylinder charge in the combustion process, are in use. Therefore, similar trends at these figures show that similar mixture formation may be assumed in all cases.

Figures 8 & 9 show the in-cylinder NOx and soot mass fractions with the different traces for spray angles and number of holes.

The results shown in Figure 5 & 6 indicate that increase spray angle and number of holes reduce the peak cylinder pressure and temperature. The lower combustion temperature resulted in reduced NOx formation. At the higher spray angle, fuel jet is injected in the center of combustion chamber and the spray may impinge on the opposing wall, and the formation of a liquid wall film is possible.

Liquid wall films usually have a negative influence on emissions, because the wall film evaporates slower and may only be partially burnt.

Therefore, the increased spray angle leads to an increase of soot formation because the soot formation took place in the spray impinging regions.

This effect is responsible for decreased NOx, increased soot and increased hydrocarbon emissions. Also with the increasing number of holes and with assumption of constant fuel per cycle values, because of fast mixing fuel and air, the

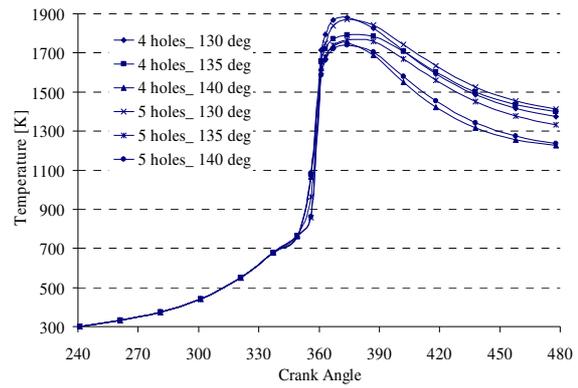


Fig.6: Comparison of temperature traces for different number of holes and spray angle at 1400rpm

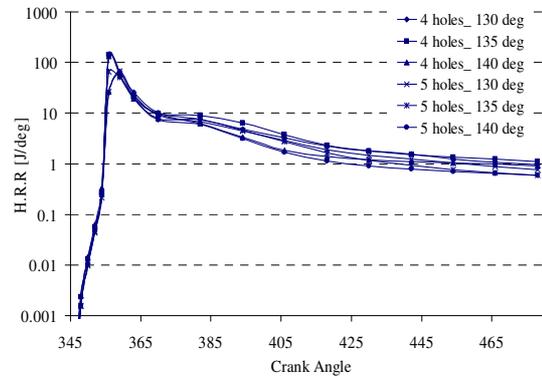


Fig.7: Comparison of heat release rate traces for different number of holes and spray angle at 1400rpm

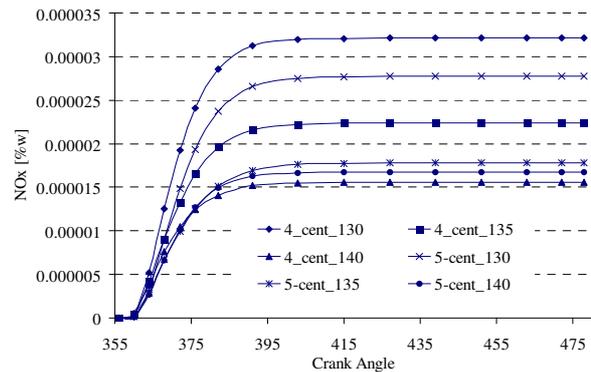


Fig.8: Comparison of NOx traces for different spray angles and number of holes

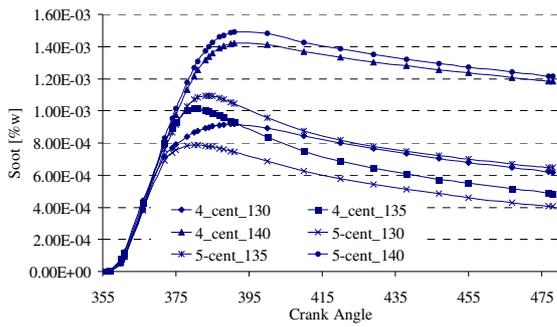


Fig.9: Comparison of soot traces for different spray angles and number of holes

amount of premixed combustion fuel burning decreases and then NOx decreases and soot increases.

Figure 10 show the in-cylinder diesel fuel mass fraction with the different traces for spray angles and number of holes.

It is clear from this figure that the trends of this figure and heat release rate curve, which is shown in figure 7, are the same. The amount of exhaust UHC is reach to maximum value at five holes with spray angle 140.

Figures 11-12 show the evolution of temperature at various spray angles (from left to right) in the symmetry plane of the engine for four and five number of holes respectively at 360°, 370°, 380° and 400°CA (from left to right). The temperature evolution reveals that the flame invades very quickly a large part of the chamber. It is clear from these figures that when spray angle increases, flame more spread out in the piston bowl than the squish regions. Similar trends at also observed at whole crank angles. At spray angle 130°, we observe flame front spread out in the center of piston bowl. This explains that spray more interacts with the flow filed at this spray angle. Therefore, fuel vapor has already been carried along by the swirl for some way and stoichiometric mixture has formed in the center of piston bowl.

The comparison of these figures shows that the increasing of the number of holes doesn't strongly effect on the flame temperature and flame spread out.

The region resolved global mass fraction of NO for the different spray angles and number of holes are given in Figures 13-15 at 370°CA,

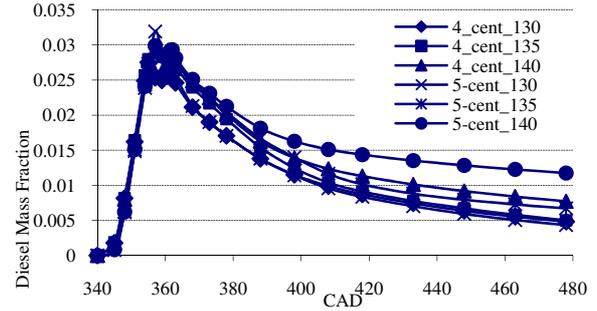


Fig.10: Comparison of UHC traces for different spray angles and number of holes

380°CA and 400°CA respectively. The NO formation starts off at about 18° CA after the start of injection. The initial increase in the global NO mass fraction follows the global temperature (fig. 6).

The comparison of these figure and figures 11-12 (flame distribution), it can be deduced that the quality of the correlation of temperature (flame) and NO-formation to the air fuel ratio at the regions with AFR=1 and temperature is higher than 2000K, the NOx product at the 370°CA,380°CA and 400°CA. However, the NOx emission decreases with high spray angle and number of holes, because of the volume of these regions decreases in these states.

Figures 16-18 show that soot is produced in regions of high fuel concentrations, when cold fuel is injected into areas of hot gases. The soot then oxidizes again in the leaner regions of the flame, so that close to the stoichiometric contour most of the soot is already consumed.

The results shown in these figures and figure 11-12 indicate that the higher spray angle and the number of holes reduces the peak cylinder pressure and temperatures. The lower combustion temperature resulted in reduced NOx and increased soot formation. At the regions with AFR>3 and temperature is near 1700K, the soot product at 370°CA, 380°CA and 400°CA. However, the soot emission increases with high spray angle and number of holes, because of the volume of these regions increases at these states. When spray angle increase, the region of the soot formation transfer form squish region to center of piston bowl. At various numbers of holes, there are similar soot distributions.

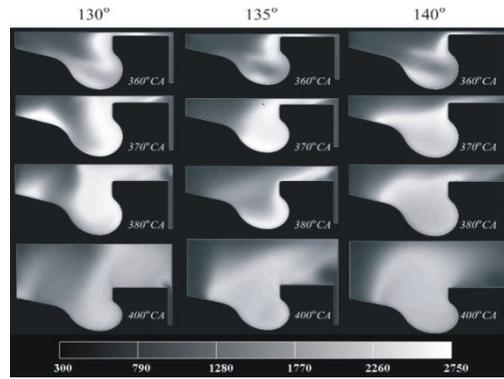


Fig.11: Temperature contours for different spray angles at four numbers of holes

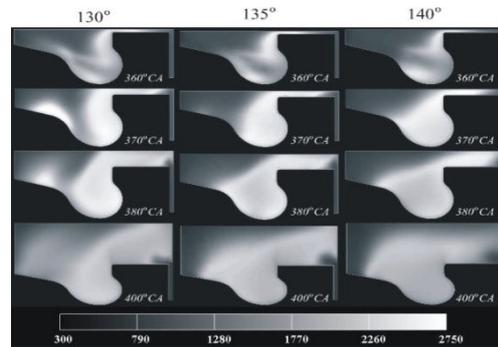


Fig.12: Temperature contours for different spray angles at five numbers of holes

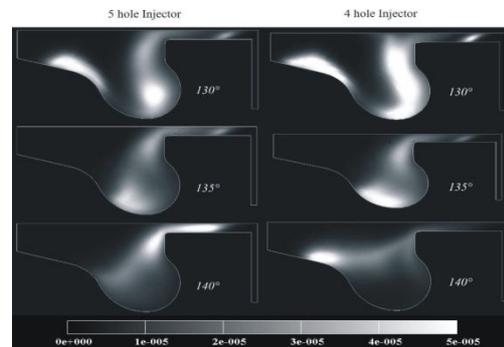


Fig.13: NOx contours for different spray angles and number of holes at 370°CA

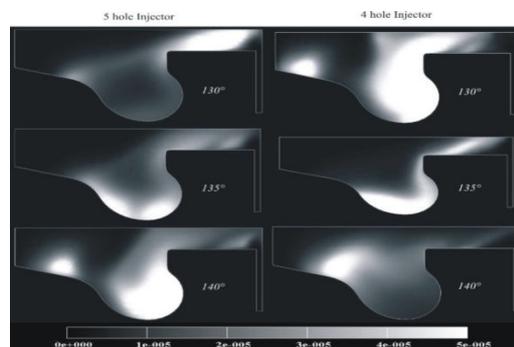


Fig.14: NOx contours for different spray angles and number of holes at 380°CA

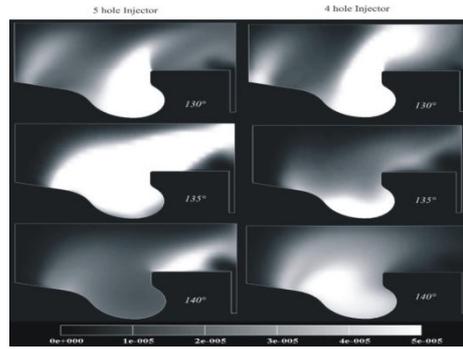


Fig.15: NOx contours for different spray angles and number of holes at 400°C

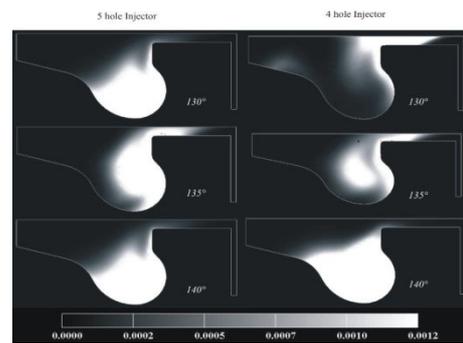


Fig.16: Soot contours for different spray angles and number of holes at 370°C

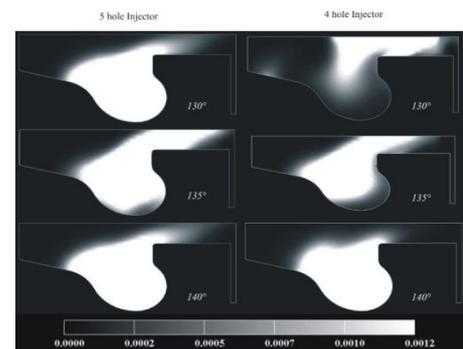


Fig.17: Soot contours for different spray angles and of holes at 380°C

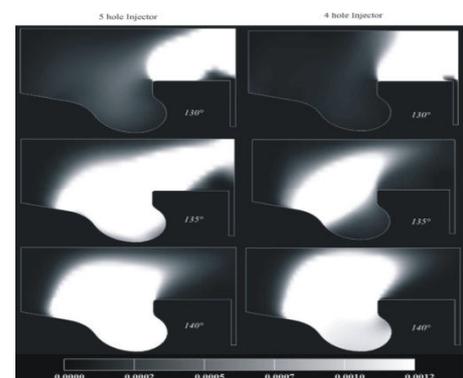


Fig.18: Soot contours for different spray angles and number of holes at 400°C

Table 2. Exhaust Emission ENGINE AT The Various Spray Angles And Number Holes

NO <sub>x</sub> [%w]	Soot [%w]		Spray angles	
	exhaust	maximum		
0.0000321	0.000617	0.000917	130°	Four holes
0.0000224	0.000485	0.001010	135°	
0.000155	0.001183	0.001418	140°	
0.0000277	0.000405	0.000786	130°	Five holes
0.0000178	0.000647	0.001092	135°	
0.0000167	0.001214	0.001497	140°	

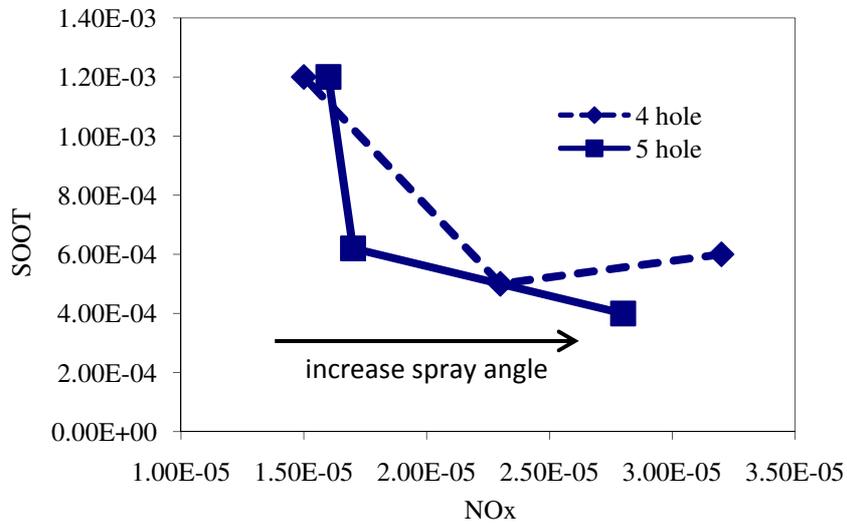


Fig.19: Soot-NOx trade off for different spray angles and number of holes

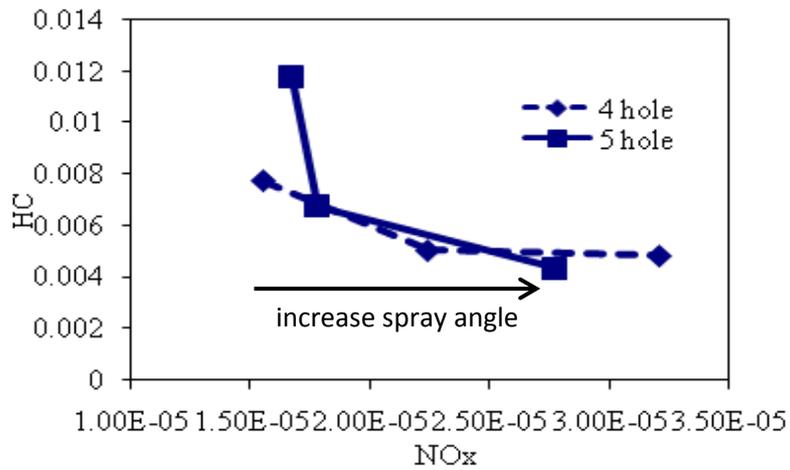


Fig. 20: UHC-NOx trade off for different spray angles and number of holes

Table 2 shows the values of computational emission at the beginning of exhaust stroke. Table 4 shows that the NOx emission decreased and soot emission increased with increasing spray angle and the number of holes simultaneously.

Figures 19 and 20 show the UHC, NOx and Soot, NOx trade-offs. The effect of number hole and spray angle was investigated using three different spray angles 130, 135, 140 and two different 4 and 5 number holes. It can be seen that the similar trends is observed in these figures. Soot and UHC decreased significantly at small spray angle and five number holes, whereas slowly decreased at four number holes. It is interesting to see that the five holes with spray angle 135 is the best point curve for soot and NOx trade off.

## 8. Conclusions

In this paper, a computational study was carried out to investigate the effects of spray angles with the number of holes in a direct injection diesel engine. Based on this study, the following conclusions are drawn:

1- With the increasing of spray angle and number of hole, peak pressure and temperature in cylinder decrease.

2- Minimum NOx emission for four hole injection system take place at spray angle corresponding to 140° and minimum soot exhaust emission for five hole injection system take place at spray angle corresponding to 130°.

3- At five hole injection system with spray angle corresponding to 130°, the amount of soot can be reduce and the amount of NOx have average value at this point. Therefore spray angle corresponding to 130° is optimum state.

4- When spray angle increases, the flame front and NOx formation regions and soot formation reach to the piston bowl from squish regions.

5- Similar trends for flame front, NOx formation regions and soot formation are observed at four and five holes.

6- Five holes with spray angle 135 because of closer to origin coordinate is the best point for soot and NOx trade off.

## 9. Nomenclature

### Greek symbols

$\Lambda(m)$  wave length

$\alpha(w/m^2k)$  heat transfer coefficient

$\tau (s)$  time scale

### Letters

$A$  pre exponential factor, area

$\dot{Q}(w)$  heat flux

$d(m)$  diameter

$\dot{r} (kg/s)$  fuel consumption rate

$E(kgm^2/s^2)$  energy

$S$  stoichiometric; source term

$K(m^3/mol.s)$  reaction constant

$T(K)$  temperature

$L(J/kg)$  latent heat of evaporation

$R(KJ/mol.k)$  universal gas constant

$M(kg/kmol)$  molecular weight

$m(kg)$  mass

$p(pascal)$  pressure

### Subscripts

$a$  activation

$form$  formation

$l$  liquid

$s$  soot, surface

$c$  carbon, critical

$fu$  Fuel

$d$  droplet

$f$  forward

$oxid$  oxidation

$fv$  fuel vapour

$R$  reaction

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